Course II

GROUP (K): GRAPH THEORY

Total Marks: 100 (End-Semester: 60 & In-Semester: 40)

INTRODUCTION:

This course describes graph theory terminologies and problems, and the use of algorithms in both mathematical theory of graphs and their applications.The course will explain in detail the basic theory of various types of graphs.This course introduces some of the algorithms that solve all or part of the problems in graph theory.

COURSE OBJECTIVE:

1. To understand and apply the basic concepts of graph theory.
2. Application of graph theory-based tools to solve practical problems.

LEARNING OUTCOME:

1. Students can apply the concepts and principles of graph theory in practical situations.

DETAILED SYLLABUS:

Unit I: Fundamental Concepts of Graph

Marks: 20

Basic definitions of graphs and multigraphs; adjacency matrices, isomorphism, girth, decompositions, independent sets and cliques, graph complements, vertex coloring, chromatic number, important graph like cubes and the Petersen graph - Paths, cycles, and trails; Eulerian circuits - Vertex degrees and counting; large bipartite subgraphs, the handshake lemma, Havel-Hakimi Theorem - Directed graphs: weak connectivity, connectivity, strong components - Induction and other fundamental proof techniques.

Unit II: Trees

Marks: 20

Basics: equivalent characterizations of trees, forests - Spanning trees and 2-switches - Distance and center - Optimization: Kruskal's Theorem and Dijkstra's Theorem - Matching and covering: Bipartite matching, vertex cover, edge cover, independent set, M-alternating path, Hall's Theorem, König-Egeváry Theorem, Gallai's Theorem.

Unit III: Connectivity

Marks: 10

Vertex cuts, separating sets, bonds; vertex and edge connectivity, block-cutpoint tree - Menger's Theorem: undirected vertex and edge versions - Network flow: Ford-Fulkerson Labeling algorithm, flow integrality, Max-flow/Min-cut Theorem, proof of Menger's Theorem – Coloring: Chromatic number: lower bounds from clique number and maximum independent set, upper bounds from greedy coloring (& Welsh-Powell), Szekeres-Wilf, and Brooks' Theorem. Also k-critical graphs, cartesian product of graphs, and interval graphs - k-Chromatic graphs: Mycielski's construction, Turán's Theorem - Edge coloring, line graphs, Vizing's Theorem

Unit IV: Planarity

Marks: 10

Embeddings, dual graphs, Euler's formula - Kuratowski's Theorem - Coloring, including the 5-color theorem.

REFERENCE BOOKS:

1. N.Deo, "Graph Theory with Application to Engineering and Computer Science", Prentice-Hall of India Pvt.Ltd, 2003.

2. L.R.Foulds , "Graph Theory Applications", Springer ,2016.